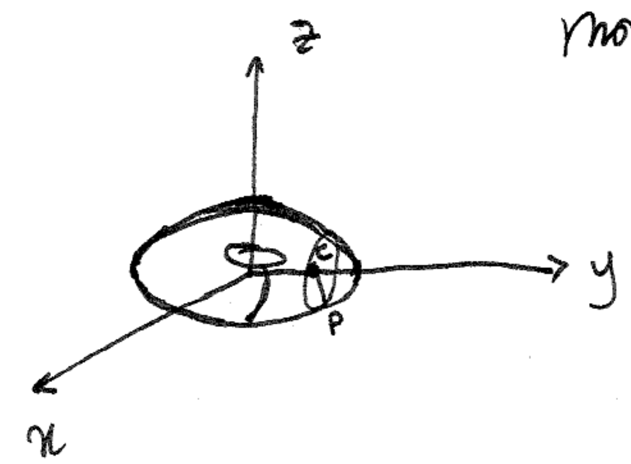
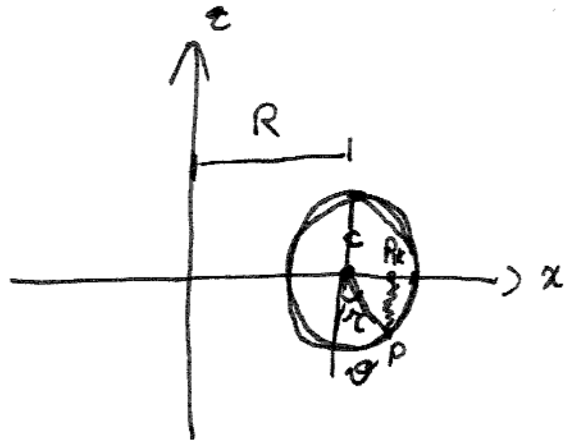
Esame 12/06/2012

massa asta m





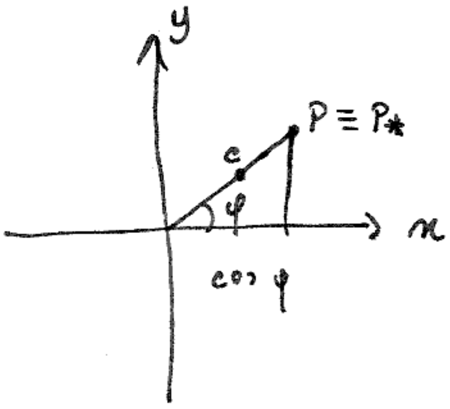
$P = (x,y,z)$ ~~$\tfrac{1}{2}R+r$~~ $C = (R\cos\varphi, R\sin\varphi, 0)$

sul piano Oxz ho che

$\overline{PC}^2 = r^2$; dato da

~~$(x - R\cos\varphi)^2 = r^2$~~

~~$x^2 + R^2\cos^2\varphi - 2xR\cos\varphi = r^2$~~



$P_* = ((R + r\sin\vartheta)\cos\varphi, (R + r\sin\vartheta)\sin\varphi, 0)$

$P = ((R+r\sin\vartheta)\cos\varphi, (R+r\sin\vartheta)\sin\varphi, -r\cos\vartheta)$

B = baricentro $= \left(\left(R+\frac{r}{2}\sin\vartheta\right)\cos\varphi, \left(R+\frac{r}{2}\sin\vartheta\right)\sin\varphi, -\frac{r}{2}\cos\vartheta\right)$

~~$\frac{(R+2r\sin\vartheta)\cos\varphi}{2}$~~

$T = \frac{1}{2} m \underline{v}_B \cdot \underline{v}_B + \frac{1}{2} I_{asse\, y} \dot\vartheta^2$ l'asta ruota intorno al solo asse y mentre nell'altro moto sono traslatorie

1

$$\underline{v}_B = \left(-\dot\varphi \sin\varphi \left(R + \frac{r}{2}\sin\vartheta\right) + \frac{r}{2}\dot\vartheta\cos\vartheta\cos\varphi,\ \dot\varphi\cos\varphi\left(R+\frac{r}{2}\sin\vartheta\right) + \frac{r}{2}\dot\vartheta\cos\vartheta\sin\varphi,\ \frac{r}{2}\dot\vartheta\sin\vartheta \right)$$

$$I_{asy} = \delta \int_{-\frac{r}{2}}^{\frac{r}{2}} d\xi\, \xi^2 = \overset{\frac{m}{r}}{\delta}\, 2\frac{r^3}{24} = \frac{mr^2}{12}$$

$$\underline{v}_B \cdot \underline{v}_B = \dot\varphi^2 \sin^2\varphi \left(R + \frac{r}{2}\sin\vartheta\right)^2 + \frac{r^2}{4}\dot\vartheta^2\cos^2\vartheta\cos^2\varphi - \cancel{r\dot\vartheta\dot\varphi\cos\varphi\sin\varphi\cos\vartheta\left(R+\frac{r}{2}\sin\vartheta\right)}$$
$$+\dot\varphi^2\cos^2\varphi\left(\quad\right)^2 + \frac{r^2}{4}\dot\vartheta^2\cos^2\vartheta\sin^2\varphi + \cancel{r\dot\vartheta\dot\varphi\cos\varphi\sin\varphi\cos\vartheta\left(\quad\right)}$$
$$+\frac{r^2}{4}\dot\vartheta^2\sin^2\vartheta = \dot\varphi^2\left(R+\frac{r}{2}\sin\vartheta\right)^2 + \frac{r^2}{4}\dot\vartheta^2\cos^2\vartheta + \frac{r^2}{4}\dot\vartheta^2\sin^2\vartheta =$$
$$= \left(R + \frac{r}{2}\sin\vartheta\right)^2\dot\varphi^2 + \frac{r^2}{4}\dot\vartheta^2$$

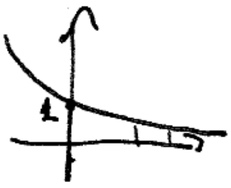
$$T = \frac{1}{2} m\left[\left(R + \frac{r}{2}\sin\vartheta\right)^2\dot\varphi^2 + \frac{r^2}{4}\dot\vartheta^2\right] + \frac{m r^2}{24}\dot\vartheta^2$$

$g = 0$

$\mathcal{E}\, e_y$ campo elettrico $\qquad -\nabla U_{el} = q\mathcal{E}\, e_y \qquad U_{el} = -\int q\mathcal{E}\, dy = -q\mathcal{E}y$

$$U = U_{el} + \frac{1}{2}k\|\overrightarrow{PP_*}\|$$

$x^{-x} \qquad \sum x^k = \frac{1}{1-x}$



$$L = T - U =$$

$$= \frac{1}{2}m\left[\left(R+\frac{r}{2}\sin\vartheta\right)^2\dot\varphi^2 + \frac{r^2}{4}\dot\vartheta^2\right] + \frac{1}{16}mr^2\dot\vartheta^2 \mp q U_{el} - \frac{1}{2}k\|\overrightarrow{PP_*}\|^2 =$$
$$= \frac{1}{2}m\left[\left(R+\frac{r}{2}\sin\vartheta\right)^2\dot\varphi^2 + \frac{r^2}{4}\dot\vartheta^2\right] + \frac{mr^2}{24}\dot\vartheta^2 + q\mathcal{E}(R + r\sin\vartheta)\sin\varphi - \frac{1}{2}k r^2\cos^2\vartheta =$$
$$= \frac{1}{2}m\left(R+\frac{r}{2}\sin\vartheta\right)^2\dot\varphi^2 + \frac{1}{6}mr^2\dot\vartheta^2 + q\mathcal{E}(R + r\sin\vartheta)\sin\varphi - \frac{1}{2}k r^2\cos^2\vartheta$$

$$\cancel{\frac{d}{dt}\frac{\partial L}{\partial \dot\vartheta}} = \cancel{\frac{m}{4} r^2 \ddot\vartheta} + \cancel{\frac{1}{6} m r^2 \ddot\vartheta} \Rightarrow$$

$$\Rightarrow \frac{\partial}{\partial \dot\vartheta} L = 2 \cdot \frac{1}{2} m \frac{r^2}{4} \dot\vartheta + \frac{m r^2 \dot\vartheta}{12} =$$

$$= \frac{1}{3} m r^2 \dot\vartheta \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot\vartheta} = \frac{1}{3} m r^2 \ddot\vartheta$$

$$\frac{\partial L}{\partial \vartheta} = \dot\varphi^2 m \left(R + \frac{r}{2}\sin\vartheta\right)\cos\vartheta + q\mathcal{E} r \cos\vartheta \sin\varphi + \frac{1}{2} k r^2 \cos\vartheta \sin\vartheta$$

$$\frac{\partial L}{\partial \dot\varphi} = m\left(R + \frac{r}{2}\sin\vartheta\right)^2 \dot\varphi \qquad \frac{d}{dt}\frac{\partial L}{\partial \dot\varphi} = \ddot\varphi\, m\left(R + \frac{r}{2}\sin\vartheta\right)^2 + \dot\varphi\, m \cancel{2}\left(R + \frac{1}{2}\sin\vartheta\right)\frac{r}{\cancel{2}}\cos\vartheta\,\dot\vartheta =$$

$$= m\left(R + \frac{1}{2}\sin\vartheta\right)^2 \ddot\varphi + m r \dot\varphi \dot\vartheta \left(R + \frac{1}{2}\sin\vartheta\right)$$

$$\frac{\partial L}{\partial \varphi} = q\mathcal{E}(R + r\sin\vartheta)\cos\varphi$$

$$\begin{cases} \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot\vartheta} - \dfrac{\partial L}{\partial \vartheta} = \dfrac{1}{3} m r^2 \ddot\vartheta - q\mathcal{E} r \cos\vartheta \sin\varphi - k r^2 \cos\vartheta \sin\vartheta - m\dot\varphi^2 \cos\vartheta \left(R + \frac{3}{2}\sin\vartheta\right) = 0 \\ \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot\varphi} - \dfrac{\partial L}{\partial \varphi} = m\left(R + \frac{1}{2}\sin\vartheta\right)^2 \ddot\varphi + m r \dot\varphi \dot\vartheta \left(R + \frac{1}{1}\sin\vartheta\right) - q\mathcal{E}(R + r\sin\vartheta)\cos\vartheta = 0 \end{cases}$$

Punti di equilibrio

$$\begin{cases} 1) \quad \dfrac{\partial U}{\partial \vartheta} = -q\mathcal{E} r \cos\vartheta \sin\varphi - k r^2 \cos\vartheta \sin\vartheta = \cancel{r}\cos\vartheta\left(+q\mathcal{E}\sin\varphi + k r \sin\vartheta\right) = 0 \\ 2) \quad \dfrac{\partial U}{\partial \varphi} = -q\mathcal{E}(R + r\sin\vartheta)\cos\varphi = 0 \end{cases}$$

oss $\quad R > r > 0 \Rightarrow R > r\sin\vartheta$

2) $\quad \cos\varphi = 0 \quad \vee \quad \sin\vartheta = -\frac{R}{r}$ ma $-\frac{R}{r} < -1 \Rightarrow$ NA

$\iff \varphi = \pm\frac{\pi}{2}$

- $\varphi = \pm\frac{\pi}{2} \Rightarrow$ sono sull'asse y
- $\varphi = \frac{\pi}{2} \Rightarrow$ sono su $y > 0 \Rightarrow \sin\varphi = 1$

1) $\quad \cos\vartheta\,(q\mathcal{E} + k r \sin\vartheta) = 0$

$\cos\vartheta = 0 \quad \vee \quad \sin\vartheta = -\frac{q\mathcal{E}}{kr}$

$\Updownarrow \qquad\qquad \Updownarrow$

$\vartheta = \pm\frac{\pi}{2} \qquad \vartheta = \arcsin\left(-\frac{q\mathcal{E}}{kr}\right) = -\beta \quad \vee \quad \vartheta = \beta + \pi$

$\cancel{\mathrm{Hess}\,U} = \quad \partial_\vartheta U = -q\mathcal{E}r\cos\vartheta\sin\varphi - kr^2\cos\vartheta\sin\vartheta =$

$= -q\mathcal{E}r\cos\vartheta\sin\varphi - kr^2\dfrac{\sin 2\vartheta}{2}$

$$\mathrm{Hess}\,U = \begin{pmatrix} q\mathcal{E}r\sin\vartheta\sin\varphi - \cancel{2}k\frac{r^2}{\cancel{2}}\cos 2\vartheta & -q\mathcal{E}r\cos\vartheta\cos\varphi \\ -q\mathcal{E}r\cos\vartheta\cos\varphi & q\mathcal{E}(R+r\sin\vartheta)\sin\varphi \end{pmatrix}$$

$- \quad \varphi = \frac{\pi}{2} \quad A = \mathrm{Hess}\,U = \begin{pmatrix} q\mathcal{E}r\sin\vartheta - \cancel{2}kr^2\cos 2\vartheta & 0 \\ 0 & q\mathcal{E}(R+r\sin\vartheta) \end{pmatrix}$

$\vartheta = \frac{\pi}{2} \qquad A = \begin{pmatrix} q\mathcal{E}r + \cancel{2}kr^2 & 0 \\ 0 & q\mathcal{E}(R+r) \end{pmatrix}$

$\det A = \underbrace{q\mathcal{E}(R+r)}_{>0}\left[q\mathcal{E}r + \underbrace{\cancel{2}kr^2}_{>0}\right] > 0 \quad \to \lambda_1 > 0$

$\mathrm{Tr}\,A = \underbrace{q\mathcal{E}r}_{>0} + \underbrace{\cancel{2}kr^2}_{>0} + \underbrace{q\mathcal{E}(R+r)}_{>0} \quad > 0 \to \lambda_2 > 0$

$(q\mathcal{E}r + 2kr^2 - \lambda)(q\mathcal{E}(R+r) - \lambda) = \lambda^2 - (q\mathcal{E}r + \cancel{2}kr^2)\lambda - q\mathcal{E}(R+r)\lambda +$
$+ q^2\mathcal{E}^2 r(R+r)$

$$\lambda_{1,2} = \frac{q\mathcal{E}r + 2kr^2 - q\mathcal{E}(R+r) \pm \sqrt{q^2\mathcal{E}^2r^2 + 4k^2r^4 + q^2\mathcal{E}^2(R+r)^2 + q^2\mathcal{E}^2r(R+r) + 2q\mathcal{E}kr^2 + 2q\mathcal{E}kr^2(R+r) - 4q^2\mathcal{E}^2r(R+r)}}{2}$$

$\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$ è min. $\Rightarrow$ STAB

BOH

$-\ \varphi = \frac{\pi}{2} \qquad \vartheta = -\frac{\pi}{2}$

$$A = \begin{pmatrix} -q\varepsilon r + \cancel{\gamma} k r^2 & 0 \\ 0 & q\varepsilon(R-r) \end{pmatrix}$$

$>0 \ (R>r)$

$$\det A = r q\varepsilon(R-r)(\cancel{\gamma} k r - q\varepsilon) \gtreqless 0 \iff \frac{q\varepsilon}{kr} \lesseqgtr 1$$

$kr - q\varepsilon > 0 \ ; \quad kr > q\varepsilon \qquad \frac{q\varepsilon}{kr} < 1$

$\frac{q\varepsilon}{kr} < 1 \qquad \lambda_{1,2} > 0 \qquad$ STAB

$\frac{q\varepsilon}{kr} > 1 \qquad \lambda_1 \cdot \lambda_2 < 0 \qquad$ INST

$\frac{q\varepsilon}{kr} = 1 \qquad \lambda_1 = 0 \quad \lambda_2 > 0 \qquad$ BOH

$-\ \varphi = \frac{\pi}{2} \qquad \vartheta = -\beta$

$$\cos 2\vartheta = \frac{e^{i2x} + e^{-i2x}}{2} = \frac{(\overset{A}{e^{ix}})^2 + (\overset{B}{e^{-ix}})^2}{2} = \frac{A^2+B^2}{2} = \frac{A^2+B^2+2AB}{2} - AB = \frac{(A+B)^2}{2} - AB = 2\left(\frac{e^{ix}+e^{-ix}}{2}\right)^2 - 1 = 2\cos^2\vartheta - 1 = \cancel{1} - 2\sin^2\vartheta \ \cancel{+}$$

$$\Downarrow$$

$$\operatorname{Hess} U\big|_{\varphi=\frac{\pi}{2}} = \begin{pmatrix} q\varepsilon r\sin\vartheta - kr^2 + 2kr^2\sin^2\vartheta ; 0 \\ 0 ; q\varepsilon(R + r\sin\vartheta) \end{pmatrix}$$

$$A: \operatorname{Hess} U\big|_{\substack{\vartheta=-\beta \\ \varphi=\frac{\pi}{2}}} = \begin{pmatrix} -q^2\varepsilon^2 \frac{1}{kr} - kr^2 + 2kr^2\frac{q^2\varepsilon^2}{k^2r^2} ; 0 \\ 0 \quad q\varepsilon\left(R + \frac{q\varepsilon}{k}\right) \end{pmatrix} = \begin{pmatrix} \frac{1}{k}(q^2\varepsilon^2 - k^2r^2) & 0 \\ 0 & q\varepsilon\left(R + \frac{q\varepsilon}{k}\right) \end{pmatrix}$$

>0

$$1)\ \det A = \underbrace{\frac{1}{k}}_{>0}(q^2\varepsilon^2 - k^2r^2)\underbrace{q\varepsilon}_{>0}\underbrace{\left(R + \frac{q\varepsilon}{k}\right)}_{>0} \gtrless 0$$

$$\operatorname{Tr} A = \frac{1}{k}(q^2\varepsilon^2 - k^2r^2) + q\varepsilon\left(R + \frac{q\varepsilon}{k}\right) \gtrless 0$$

1) $q^2\varepsilon^2 - k^2r^2 > 0 \qquad q^2\varepsilon^2 > k^2r^2 \qquad \frac{q^2\varepsilon^2}{k^2r^2} > 1 \qquad \frac{q\varepsilon}{kr} > 1 \qquad$ IMP!

$q^2\varepsilon^2 - k^2r^2 < 0 \qquad \frac{q\varepsilon}{kr} < 1 \qquad \Rightarrow$ sentinel $\lambda_1 < 0$

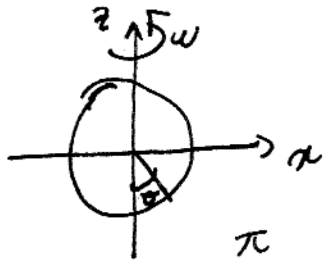
$= 0 \qquad \frac{q\varepsilon}{kr} = 1 \qquad \lambda_1 = 0$

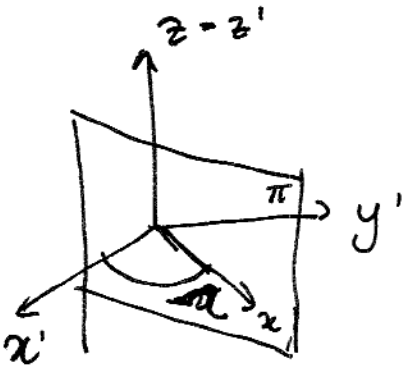
II eq. cardinale per l'asta

M rispett. a Q

caso (0,0)

$$\dot{M} = \sum_j \overrightarrow{QP_j} \wedge F_j - v_Q \wedge P$$





oss forze apparenti

$-m\underline{a}_t = m\,\Omega^2\,\overrightarrow{P_* P}$

centrif. allontana dal piano, invece ho reazione

$\omega \wedge \underline{v}'_P \parallel \underline{e}_y \ (\perp \pi)$

$-e^{-x}\ ,\ e^{-x}$

$\cancel{\Sigma}\ \frac{(-x)}{k!}$

$$f(x) = f(0) + \frac{f'(0)}{1}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3$$

$$e^{-x} = 1 + \frac{-1}{1}x + \frac{1}{2}x^2$$

$$= 1 - x$$

$$\frac{1}{\cancel{k}}\left(q^2\varepsilon^2 - k^2 r^2\right) + \cancel{k}q\varepsilon\underbrace{\left(Rk + \frac{q\varepsilon}{\cancel{k}}\right)}_{>0} > 0$$

$\frac{q\varepsilon}{kr} > 1$ IMP

$$q^2\varepsilon^2 - k^2 r^2 + q\varepsilon Rk + q^2\varepsilon^2 > 0$$

$$2q^2\varepsilon^2 + q\varepsilon Rk - k^2r^2 = 2q^2\varepsilon^2 + k(q\varepsilon R - kr^2) > 2q^2\varepsilon^2 + k(q\varepsilon R - q\varepsilon r) =$$

$q\varepsilon < kr$

$$= \underset{>0}{2q^2\varepsilon^2} + \underset{>0}{q\varepsilon k}\underset{>0}{(R-r)} > 0$$

dunque tu portim per $\frac{q\varepsilon}{kr} < 1$ $\Rightarrow$ $\lambda_1 < 0$, $\lambda_2 > 0$ $\Rightarrow$ $(\sim \delta T$

$q\varepsilon = kr$

$$\frac{1}{k}\underbrace{\left(q^2\varepsilon^2 - k^2r^2\right)}_{=0} + \underset{>0}{q\varepsilon}\left(R + \underset{>0}{\frac{q\varepsilon}{k}}\right) > 0 \Rightarrow \text{BOH}$$

$- \ \vartheta = \pi + \beta \quad \varphi = \frac{\pi}{2}$

$$\mathrm{Hess}\, U\big|_{\varphi=\frac{\pi}{2}} = \begin{pmatrix} q\varepsilon r \sin\vartheta - kr^2 + 2kr^2\sin^2\vartheta & 0 \\ 0 & q\varepsilon(R + r\sin\vartheta) \end{pmatrix}$$

$$A = \mathrm{Hess}\, U\big|_{\substack{\varphi=\frac{\pi}{2} \\ \vartheta=\pi+\beta}} = \begin{pmatrix} \frac{1}{k}\left(q^2\varepsilon^2 - k^2r^2\right) & 0 \\ 0 & q\varepsilon\left(R + \frac{q\varepsilon}{k}\right) \end{pmatrix}$$

$$\sin(\pi+\beta) = \frac{e^{i(\pi+\beta)} - e^{-i(\pi+\beta)}}{2i} = \frac{e^{i\pi}e^{i\beta} - e^{-i\pi}e^{-i\beta}}{2i} = \frac{e^{i(-\beta)} - e^{-i(-\beta)}}{2i} = \sin(-\beta)$$

è uguale al caso precedente!

Caso $\varphi = -\frac{\pi}{2}$

$$A(\vartheta) = \text{Hess}\, U|_{\varphi=-\frac{\pi}{2}} = \begin{pmatrix} -q\mathcal{E}r\sin\vartheta - kr^2 + 2kr^2\sin^2\vartheta & 0 \\ 0 & -q\mathcal{E}(R + r\sin\vartheta) \end{pmatrix}$$

$- \vartheta = \frac{\pi}{2}$
$$A\left(\tfrac{\pi}{2}\right) = \begin{pmatrix} -q\mathcal{E}r + kr^2 & 0 \\ 0 & -q\mathcal{E}(R+r) \end{pmatrix}$$

$$\det A\left(\tfrac{\pi}{2}\right) = (-q\mathcal{E}r + kr^2)(-q\mathcal{E}(R+r)) = r\underbrace{q\mathcal{E}(R+r)}_{>0}(q\mathcal{E} - kr) \gtreqless 0$$

$q\mathcal{E} - kr > 0 \iff \frac{q\mathcal{E}}{kr} > 1$

$= 0 \iff \frac{q\mathcal{E}}{kr} = 1$ BOH

$< 0 \iff \frac{q\mathcal{E}}{kr} < 1 \Rightarrow$ INST

$$\mathrm{Tr}\left(A\left(\tfrac{\pi}{2}\right)\right) = -q\mathcal{E}r + kr^2 - q\mathcal{E}(r+R) = \cancel{r(kr - 2q\mathcal{E}) - q\mathcal{E}R} =$$
$$= kr^2 - q\mathcal{E}(R+2r) > 0$$
$$kr - q\mathcal{E}\left(\frac{R}{r} + 2\right) > 0$$
$$\frac{kr}{q\mathcal{E}} > \frac{R}{r} + 2 \iff \frac{q\mathcal{E}}{kr} < \frac{1}{\frac{R}{r}+2} = \frac{r}{R+2}$$

ma $\frac{r}{R+2} < 1$ ~~mentre~~ $\Rightarrow$ NON È POSS CHE $\frac{q\mathcal{E}}{kr} > 1$

$\Rightarrow$ $\mathrm{Tr}\left(A\left(\tfrac{\pi}{2}\right)\right) > 0 \iff \frac{q\mathcal{E}}{kr} < 1 \iff \left(\frac{\pi}{2}, -\frac{\pi}{2}\right)$ è inst.

$\frac{q\mathcal{E}}{kr} = 1 \Rightarrow \mathrm{Tr}(A) = -\cancel{kr^2} + \cancel{kr^2} - q\mathcal{E}(r+R) < 0 \Rightarrow$ inst.

$-\beta = \arcsin\left(-\frac{q\mathcal{E}}{kr}\right)$ $\qquad$ $\sin(-\beta) = \sin\left(\arcsin\left(-\frac{q\mathcal{E}}{kr}\right)\right) = -\frac{q\mathcal{E}}{kr}$

$\sin(\pi - \beta) = -\sin\beta$

$$- \vartheta = -\frac{\pi}{2} ; A\left(-\frac{\pi}{2}\right) = \begin{pmatrix} q\mathcal{E}r + kr^2 & 0 \\ 0 & -q\mathcal{E}(R-r) \end{pmatrix}$$

$$\det A = -(q\mathcal{E}r + kr^2)\, q\mathcal{E}(R-r) < 0 \Rightarrow \text{INST.}$$

$$- \vartheta = \gamma ; A(\gamma) = \begin{pmatrix} -\frac{q^2\mathcal{E}^2}{k} - kr^2 + 2kr^2 \frac{q^2\mathcal{E}^2}{k^2r^2} & 0 \\ 0 & -q\mathcal{E}(R + r\sin\vartheta) \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{q^2\mathcal{E}^2}{k} - kr^2 & 0 \\ 0 & \underbrace{\sim}_{<0} \end{pmatrix}$$

$$\det A = -\left(\frac{q^2\mathcal{E}^2}{k} - k^2r^2\right) \cancel{q\mathcal{E}(R + r\sin\vartheta)} > 0$$

$$q^2\mathcal{E}^2 - k^2r^2 < 0$$

$$\frac{q^2\mathcal{E}^2}{k^2r^2} < 1 \qquad \frac{q\mathcal{E}}{kr} < 1 \Rightarrow \det > 0$$

$\varphi = -\frac{\pi}{2}$

$\Rightarrow$ 1ª eq. di Lagrange: $\cos\vartheta(-q\mathcal{E} + kr\sin\vartheta) = 0$

$\Leftrightarrow$

$\vartheta = \pm\frac{\pi}{2} \quad \vee \quad \sin\vartheta = \frac{q\mathcal{E}}{kr}$

$\beta = \gamma = \arcsin \frac{q\mathcal{E}}{kr}$

$$\text{Tr}(A) = \frac{q^2\mathcal{E}^2}{k} - kr^2 - q\mathcal{E}\left(R + \cancel{\frac{q\mathcal{E}}{kr}}\right) > 0$$

$$= \cancel{\frac{q^2\mathcal{E}^2}{k}} - kr^2 - q\mathcal{E}R - \cancel{\frac{q^2\mathcal{E}^2}{k}} < 0 \Rightarrow \text{NST.}$$

$$\det A = -\left(\frac{q^2\mathcal{E}^2}{k} - kr^2\right) \cancel{q\mathcal{E}(R + r\sin\vartheta)} < 0$$

$$q^2\mathcal{E}^2 - k^2r^2 > 0 \Leftrightarrow \frac{q\mathcal{E}}{kr} > 1 \quad \text{IMP.}$$

Caso $(\pi-\beta, -\frac{\pi}{2})$

$$A\left(\pi-\beta, -\tfrac{\pi}{2}\right) = \begin{pmatrix} \frac{q^2\varepsilon^2}{k} - kr^2 + 2\frac{q^2\varepsilon^2}{k} & 0 \\ 0 & -q\varepsilon(R - r\sin\beta) \end{pmatrix}$$

$\sin \pi-\beta = -\sin\beta$

 $\beta = \arcsin\frac{q\varepsilon}{kr}$

$$\mathrm{Hess}\,U = \begin{pmatrix} qr\varepsilon\sin\varphi\sin\vartheta - kr^2 + 2kr^2\sin^2\vartheta & -q\varepsilon r\cos\varphi\cos\vartheta \\ -q\varepsilon r\cos\varphi\cos\vartheta & q\varepsilon(R+r\sin\vartheta)\sin\varphi \end{pmatrix}$$

$$\mathrm{Hess}\,U\Big|_{\substack{\varphi=-\frac{\pi}{2}\\ \vartheta=\pi-\beta}} = \begin{pmatrix} +\frac{q^2\varepsilon^2}{k} - kr^2 & 0 \\ 0 & -q\varepsilon(R+r\sin\beta) \end{pmatrix} = \mathrm{Hess}\,U\Big|_{\substack{\varphi=-\frac{\pi}{2}\\ \vartheta=\beta}}$$

$$\sin(\pi-\beta) = \frac{e^{i(\pi-\beta)} - e^{-i(\pi-\beta)}}{2i} = \frac{e^{i\pi}e^{-i\beta} - e^{-i\pi}e^{i\beta}}{2i} =$$

$$= \frac{-e^{-i\beta} + e^{i\beta}}{2i} = \sin\beta$$

$\cancel{\det \mathrm{Hess}\,U\big|_{\varphi=-\frac{\pi}{2},\ \vartheta=\pi-\beta}} =$

$\Rightarrow$ ho stessi autoval

Rimangono i casi BOH:

1) $(-\frac{\pi}{2}, \frac{\pi}{2})$ $\quad \frac{q\varepsilon}{kr} < 1$

2) $(-\beta, \frac{\pi}{2})$ $\quad$ - - —

3) $(\pi+\beta, \frac{\pi}{2})$ $\quad$ - —

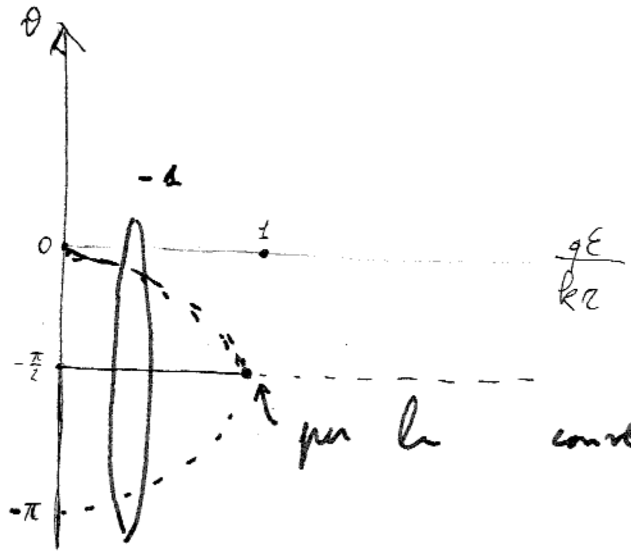
~~$U(\vartheta,\varphi) = -q\mathcal{E}(R + r\sin\vartheta)\sin\varphi + \frac{1}{2}k r^2 \cos^2\vartheta$~~

~~$U(\frac{\pi}{2},\frac{\pi}{2}) = -q\mathcal{E}(R - r)$~~ $\quad \frac{q\mathcal{E}}{kr} = 1 \Rightarrow$

~~$\Rightarrow U = -kr(R + r\sin\vartheta)\sin\varphi + \frac{1}{2}kr^2\cos^2\vartheta$~~

$U = kr\left(\frac{r}{2}\cos^2\vartheta - (R + r\sin\vartheta)\sin\varphi\right)$

Caso $\left(-\beta, \frac{\pi}{2}\right)$, $\quad \frac{q\mathcal{E}}{kr} = 1$



$\frac{q\mathcal{E}}{kr} = 1 \Rightarrow \vartheta = \beta = \arcsin(1) = \frac{\pi}{2}$

$\frac{q\mathcal{E}}{kr} = 0 \Rightarrow \vartheta = 0, \pi$

per la conversione ho INST. $\Rightarrow$ inst.

~~$\frac{q\mathcal{E}}{kr} \neq 1 \Rightarrow$ INST.~~ ho 2 casi in 1

$(-\frac{\pi}{2}, \frac{\pi}{2})$

~~eq. di Lagrange~~

~~1) $\frac{1}{3}mr^2\ddot{\vartheta} - \frac{mr^2}{2}(R - \frac{r}{2})^2 = 0$~~

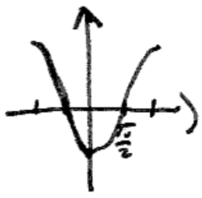
~~2) $m(R \mp \frac{r}{2})^2\ddot{\varphi} = 0$~~

$\varphi = \frac{\pi}{2}$ fisso, cost.

$$\begin{cases} \frac{1}{3}mr^2\ddot{\vartheta} - r q\mathcal{E}\cos\vartheta - kr^2\cos\vartheta = 0 \\ 0 = 0 \end{cases}$$

ho problema unidimen. con

$U' = \cancel{\sin q\mathcal{E}} - r\cos\vartheta(\underset{>0}{q\mathcal{E}} + \underset{>0}{kr}) = 0 \iff \cos\vartheta = 0 \iff \vartheta = \pm\frac{\pi}{2}$



$-\cos\vartheta \gtrless 0$

$-\pi \quad -\frac{\pi}{2} \quad 0 \quad \frac{\pi}{2} \quad \pi$

$\nearrow$ MAX $\searrow$ MIN $\nearrow$

$\Rightarrow \vartheta = -\frac{\pi}{2}$ MAX $\Rightarrow$ INST.

$\vartheta = \frac{\pi}{2}$ min $\Rightarrow$ STAB.

$\cos(\pi+\beta) = -\cos\beta$

$\sin(\pi+\beta) = -\sin\beta$

$\beta = \frac{\pi}{2} \Rightarrow \begin{cases} 0 \\ -1 \end{cases}$

(ϑ,φ) \ $\frac{g^2}{\ell k^2}$	<1	$=1$	>1
$(\frac{\pi}{2},\frac{\pi}{2})$	STAB	STAB	STAB
$(-\frac{\pi}{2},\frac{\pi}{2})$	STAB	~~BOH~~ INST (10)	INST
$(-\beta,\frac{\pi}{2})$	INST.	~~BOH~~ INST (10)	∄
$(\pi+\beta,\frac{\pi}{2})$	INST.	~~BOH~~ STAB (10)	∄
$(\frac{\pi}{2};-\frac{\pi}{2})$	INST.	INST. ~~BOH~~	INST
$(-\frac{\pi}{2};-\frac{\pi}{2})$	INST	INST	INST
$(\beta,-\frac{\pi}{2})$	INST.	INST.	∄
$(\pi-\beta,-\frac{\pi}{2})$	INST.	INST.	∄

Dubbi:

- pag. 10 caso $\psi = \frac{\pi}{2}$
  considero costante $\varphi(t)$? se sì perché?
- pag. 10 caso $\psi = \frac{\pi}{2}$
  prendo $\theta = \frac{\pi}{2}$ come soluzione della prima eq. di Lagrange o devo fare ulteriori considerazioni? se sì quali?
- calcolo delle reazioni vincolari?